Explicit model of photovoltaic panels to determine voltages and currents at the maximum power point

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ABSTRACT

A simple explicit photovoltaic formulation for characterizing and dimensioning cell-arrays is presented. The method permits the short-circuit current, the open-circuit voltage, the maximum cell power and the optimum cell-operation conditions to be determined. Further, the model also allows the easily quantifying of the effects of panel temperature and solar irradiance on key cell parameters. Based on several datasheets, the methodology is validated by covering a wide range of operating conditions. The proposed approach can thus be very useful for design engineers to determine quickly and easily the performance of any photovoltaic array without performing tedious iterative numerical calculations.

1. INTRODUCTION

Solar energy is the unique renewable (i.e., neglecting entropy creation by the Sun) resource in the world, where hydro and wind constitute secondary forms of the same source. From this perspective, it should be very convenient to directly convert solar energy into electricity by using the highest possible efficient physical process. Solar energy conversion techniques can involve thermal, electromagnetic or a combination of both forms of energy that attain our planet. The conversion of solar radiation into electricity has been extensively studied [1-5]. In particular, photovoltaic (PV) cells allow the energy transported by electromagnetic waves (i.e., photons) to be directly converted into electricity. The mechanisms that allow this energy conversion to take place are based on photon-electron interactions that occur in PN junctions formed by appropriately doped semiconductor materials. Mono-crystalline and polycrystalline silicon cells are currently found in the market [6]. In these materials, photons having enough energy are able to transfer it to covalent electrons and thus create electron-hole pairs. Even though the released electrons tend to return to their initial quantum state throughout recombination mechanisms; collecting them through an external load before recombination occurs permits an electric current to be established.

A photovoltaic panel, or array, is composed of several unitary cells connected in series and/or in parallel. Depending on the available surface area exposed to the Sun, PV panels can be employed in small and large scale applications as auxiliary electric generators in buildings and stand-alone power plants. Nowadays, this type of solar energy conversion is expanding very rapidly; consequently, predicting the performance of PV panels is essential for design engineers. Even though the most important electrical characteristics of PV panels are usually provided by manufacturers, in general, they are determined under Standard Test Conditions (STC). It is apparent, however, that under real operating conditions (i.e., varying light intensities as well as large temperature excursions) most of commercial panels do not necessary behave as given in the specifications. In particular, two factors strongly affect the overall performance of PV panels: the cell temperature and the solar irradiance. It is apparent that the last factor changes accordingly to Earth locations, time of day and seasons.

Different models based on the current vs. voltage (*I-V*) characteristic curve of a PN junction are used to describe the behavior of PV cells. In these models, a photocurrent is associated to the generation of electron-hole pairs, while a recombination current accounts for diffusion of electrons and holes across the junction. Furthermore, series and parallel electrical resistances are usually included in the models [6] to represent internal losses caused by the interconnections of cells to form arrays. The ideal PV cell model, however, considers only photocurrent and recombination current, where effects of electrical resistances are neglected [6]. Even though it is possible to introduce some simplifying assumptions on both series [7] and parallel [8,9] resistances, it is apparent that practical applications must include the effects of both of them. In addition, the PN junction itself is modeled as single- or double-diodes without resistances. Nevertheless, three-diode models can also be found in the open literature [10]. In general, double-diode models are more accurate for polycrystalline silicon cells [11] while single-diode ones are used for amorphous silicon cells [11]. It is obvious that the final model application must correspond to the best compromise between simplicity and accuracy [6].

From the characteristic *I-V* curve of a given PV cell, three key physical quantities are defined: the short-circuit current, the open-circuit voltage and the values of current and voltage that permit the maximum power to be obtained. These variables correspond to well defined points in the *I-V* plane. The determination of these points is essential to develop appropriate PV cell models. The nonlinear and implicit relationships that exist between them, however, necessitate using tedious iterative numerical calculations [6]. Furthermore, most of these parameters depend on both the cell temperature and the solar irradiance; thus, the knowledge of their behavior is crucial to correctly predict the performance of PV cells and arrays.

In general, short-circuit current, open-circuit voltage, maximum power voltage, current and power, are determined by the manufacturers under STC, i.e., irradiance of 1000 W/m², cell temperature of 25°C and AM1.5 spectrum, and under Nominal Operating Cell Temperature (NOCT) conditions, i.e., irradiance of 800 W/m², temperature of 45 or 47°C (depending on the manufacturer), ambient temperature of 20°C, wind speed of 1 m/s and AM1.5 spectrum. In some cases, they also provide temperature coefficients for the short-circuit current, the open-circuit voltage and the overall PV panel efficiency. The number of cells in series and the size of the array are also given by the manufacturers. In general, however, not all cell parameters such as the series and parallel resistances are given; thus accurate models or correlations have been developed from datasheets to determine these missing values [6,8,12-14].

Explicit expressions have been written to estimate PV efficiency, power, short-circuit current and open-circuit voltage. An excellent review of different correlations used to determine these quantities as a function of irradiance and especially cell temperature is given among others in: Skoplaki & Palyvos [15]. Experimental investigations of the influence of temperature and illumination on the *I-V* characteristic curve were performed by Radziemska & Klugmann [16] for maximum power point tracking application. Using linear relationships to characterize key PV operational points, they have determined temperature and irradiance dependent coefficients for the maximum cell output power. For relatively low variations of the irradiance, they have studied the changes of open-circuit and maximum power point voltages as a function of the cell temperature. It must be pointed out that the proposed relationships are only valid for their experimental PV module. Bellini et al. [17] proposed an explicit model for determining currents and voltages for PV points by using manufacturers' data where the electrical current changes linearly with irradiance and temperature while voltages are expressed by temperature coefficients and a correction term for the irradiance. The same correction term is added for both open-circuit and maximum power point voltages and it is calculated by subtracting the open-circuit voltage at the considered irradiance to the same value at STC. De Soto et al. [18] have compared measured data to an algebraic model that uses correlations for the current and the voltage around key operational points. These correlations accurately predict PV performance using a complex dependence of key point of currents and voltages on temperature and irradiance. Nevertheless, these relations necessitate several parameters that cannot be found in manufacturers' data. Ortiz-Rivera and Peng [19] presented an analytical model based on manufacturers' data. They have explicitly expressed the PV current and power as a function of voltage by using a shading linear factor expressed for open-circuit voltage losses for irradiances ranging from 1000 W/m² to low 200 W/m². *I-V* and *P-V* curves, among others, can then be produced without calculating explicit expressions of current and voltage at key operational points. It must be pointed out that most of the aforementioned works deal with models that are essentially based on linear temperature and irradiance relationships that require parameters which are not available from manufacturers' datasheets.

In this paper, an analytical model is proposed to characterize PV cells. The proposed expressions, based on explicit methods, allow the current and the voltage at key operational points, (i.e., in particular at the maximum power point) to be calculated using the single-diode model as a function of cell temperature, irradiance and common manufacturers' data. Further, the technique permits designers and engineers to determine different parameters of a solar panel without using iterative numerical calculations. It must be pointed out that the principal motivation of this work consists of developing a convenient calculation methodology that could help performing exergy analyses of solar panels, which are not discussed in this paper.

2. PHOTOVOLTAIC PANEL MODELS

Figure 1 shows equivalent electrical schematics based on single- and double-diode ideal PV cell models. Figure 1a corresponds to the simplest case which does not include internal electrical resistances; for such a case the *I-V* characteristic curve is given by:

$$I = I_{ph} - I_o\left[\exp\left(\frac{qV}{n\,N_s k_B T}\right) - 1\right], \tag{1}$$

where I_{ph} is the photocurrent (*A*), I_o is the saturation current (*A*), q is the absolute value of electron's charge ($-1.60217646\times10^{-19}\,C$), n represents the quality factor of the diode, N_s is the number of cells connected in series, $k_B = 1.3806503\times10^{-23}\,J/K$ is Boltzmann's constant and T is the temperature of the *p-n* junction (*K*). This temperature is generally assumed to be close enough to the temperature of the cell itself [6]. Note that when several cells are connected in parallel, they implicitly affect the values of I_{ph} and I_o; thus, the current of the array becomes the product of the number of cells in parallel by the current produced by a single cell. The second term on the right side of Equation 1 (i.e., Shockley's expression) drives the current I_d in Figure 1a.

The model shown in Figure 1a and represented by Equation 1, however, does not include internal resistances of the device; therefore they have been included in Figure 1b. The presence of series (R_s) and parallel or shunt (R_{sh}) resistances affects the *I-V* equation that should be written as:

$$I = I_{ph} - I_o\left[\exp\left(\frac{q(V + I R_s)}{n\,N_s k_B T}\right) - 1\right] - \frac{V + I R_s}{R_{sh}}\,. \tag{2}$$

Comparing Equations 1 and 2, it is apparent that the series resistance affects the output voltage while the shunt resistance reduces the available electrical current. Furthermore, since the saturation current is the result of a linear superposition of charge diffusion and recombination in the space-charge layer [11] then, in the former equations I_o can be considered as the contribution of two Shockley like terms. Thus, the *I-V* model can be expressed by:

$$I = I_{ph} - \left\{I_{od}\left[\exp\left(\frac{q(V + I R_s)}{N_s k_B T}\right) - 1\right] + I_{og}\left[\exp\left(\frac{q(V + I R_s)}{2\,N_s k_B T}\right) - 1\right]\right\} - \frac{V + I R_s}{R_{sh}}\,, \tag{3}$$

where I_{od} is the contribution of charge diffusion to I_o while I_{og} represents charge recombination in the space-charge PN layer [11]. Equation 3 is usually identified with the two-diode *I-V* model; its equivalent electrical circuit is shown in Figure 1c. Similar to the one-diode model (Eq. 2) internal series and shunt resistances affect the output voltage and current respectively.

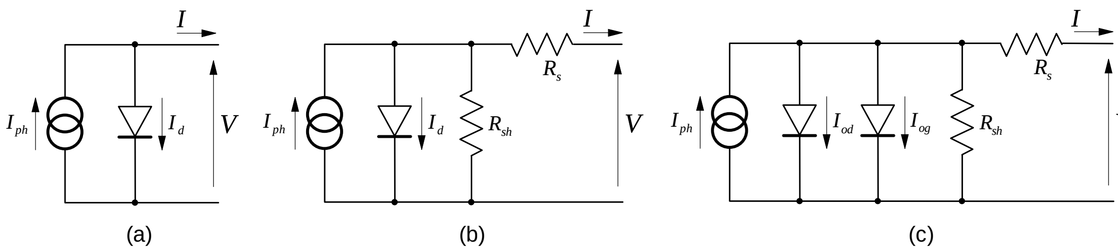

Figure 1. Equivalent electrical circuits: a) ideal PV model; b) single-diode and c) double-diode models with internal resistances.

It is a common practice to characterize PV cells as a function of three key parameters. They correspond to the following operational point conditions: the short-circuit point where $V = 0$ and $I = I_{sc}$, the open circuit point where $V = V_{oc}$ and $I = 0$, and the maximum power point where $V = V_m$ and $I = I_m$. These last values correspond to operating conditions that permit a maximum of electrical power to be achieved. This particular *I-V* state is determined from the derivative of the electrical power ($W = I.V$) with respect to the voltage, i.e., $\frac{dW}{dV} = 0$. The PV characteristic points for a cell described by Equation 1 are illustrated in Figure 2.

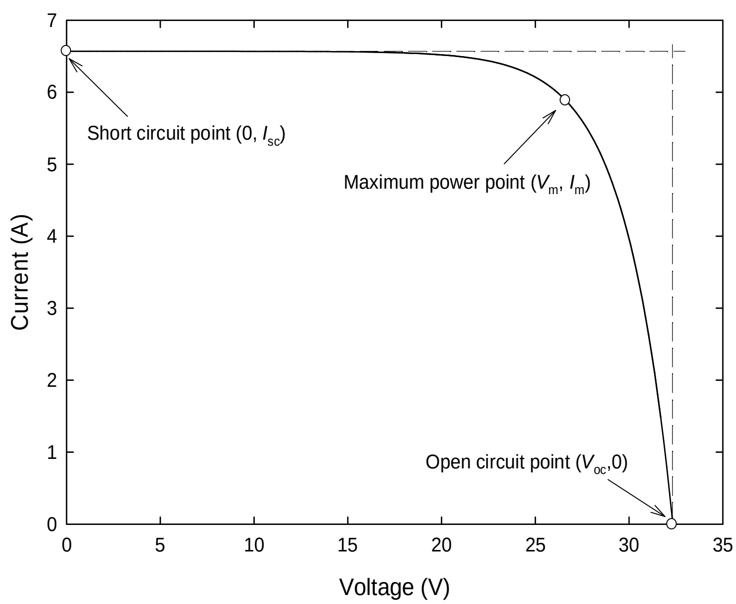

Figure 2. *I-V* characteristic curve of an ideal PV cell.

From the aforementioned models it is obvious that the PV cell acts as a current source near the short-circuit point and as a voltage source in the vicinity of the open-circuit point (Figure 1). Therefore, the series resistance R_s, which represents structural resistances of the photovoltaic panel [6], has a strong effect in the voltage-source region. In turn, the shunt resistance R_{sh} that accounts for current leakage in the PN junction [6], affects more the current-source region. Further, the maximum power point appears to be a compromise of the hybrid behavior of the cell between both voltage- and current-source regions. The quality factor of the diode is used as an adjustment parameter to account for a deviation of the ideal model (i.e., n=1 for the ideal diode model). Its value depends on the current transport mechanism [20]. A unit value describes ideal electron transport across the PN junction while a value of 2 corresponds to the superposition of diffusion and recombination mechanisms. Values higher than 2 can be obtained when multi-recombination or multi-tunneling steps occur [20]. Finally, and as was detailed before, the saturation current I_o corresponds to diffusion and recombination of carriers transported across the junction space-charge zone. It depends on the intrinsic properties of semiconductors, i.e., diffusion coefficient for the electrons, lifetime and density of intrinsic carriers [6,12]. The individual effects of each of these parameters on the characteristics of PV cells are given among others in [12].

In the aforementioned models, series and parallel resistances, in addition to the quality factor of the diode are supposed as constants and they are calculated at STC. However, Eikelboom and Reinders [21] have stipulated that these parameters are strongly affected by the irradiance because the parallel resistance decreases and the series resistance increases with increasing irradiance. They experimentally studied the change of these resistances when the irradiance is varied from about 100 to 1000 W/m². For four multi-crystalline silicon photovoltaic modules they have obtained series resistance varying from 0 to up to 0.3 **Ω**, while the parallel resistance decreases from about 600 to 300 **Ω**. In a similar study, Bätzner et al. [20] experimentally investigated the performance of HVE CdTe/CdS solar cells in comparison to conventional c-Si and GaAs solar cells. They used 4 cm² c-Si solar cells with irradiance intensities increasing from 2 to 600 W/m², they have determined series and shunt resistance changes from 0.25 to 1 Ωcm² and 4000 to 300000Ωcm² respectively. They have argued that the photoconductivity of the cells is the principal cause of the high value of the parallel resistance at low irradiance. They also studied the diode quality factor n for the c-Si cells; they have observed that it decreases from about 1.6 to 1.1 with increasing irradiance within the same range. It is obvious that the use of constant parameters determined under STC conditions must bring about the underestimation of cell efficiencies at low irradiance levels [21].

3. PROPOSED PHOTOVOLTAIC PANEL MODEL

An explicit set of equations is written based on the ideal PV model given by Equation 1. A single-diode without series and shunt resistances is considered, however, the effect of neglecting them is studied in detail in Section 4. Equation 1 is used to write down expressions for currents and voltages at each key point shown in Figure 1. Hence, the short-circuit current, the open-circuit voltage, the maximum power voltage and current are written as:

$$I_{sc} = I_{ph}\big|_{V=0}, \tag{4}$$

$$V_{oc} = \frac{n N_s k_B T}{q} \ln\left(1 + \frac{I_{sc}}{I_o}\right), \tag{5}$$

$$\exp\left(\frac{qV_{oc}}{n N_s k_B T}\right) = \left(1 + \frac{qV_m}{n N_s k_B T}\right)\exp\left(\frac{qV_m}{n N_s k_B T}\right), \tag{6}$$

$$I_m = I_{ph} - I_o\left[\exp\left(\frac{qV_m}{n N_s k_B T}\right) - 1\right]. \tag{7}$$

It is obvious that Equation 6 is implicit, therefore to obtain an explicit expression for every PV key parameter this equation has to be rewritten in a different form. As has been previously mentioned, a PV cell has a hybrid behavior, i.e., a current-source at the short-circuit point and voltage-source at the open-circuit point. These two regions are characterized by two asymptotes of the *I-V* curve in Figure 1, where the transition is a compromise between the two behaviors. It is interesting to remark that the maximum power point corresponds to a trade-off condition where the current is still high enough before it starts decreasing with increasing the output voltage (Figure 1). Based on this observation, the tangent of the *I-V* curve can be used to evaluate the transition between current- to voltage-source controlled regions; this operation yields:

$$\frac{dI}{dV} = -\frac{q\,I_o}{n\,N_s k_B T}\exp\left(\frac{qV}{n\,N_s k_B T}\right). \tag{8}$$

This derivative is then used to calculate the output voltage that corresponds to the maximum power operation condition of the cell; thus:

$$V_m = \frac{n\,N_s k_B T}{q}\ln\left(-\frac{n\,N_s k_B T}{q\,I_o}\left(\frac{dI}{dV}\right)_{V_m}\right). \tag{9}$$

It is apparent that this equation requires an expression of the derivative of the current with voltage evaluated at the maximum power point. The fact that the maximum power corresponds to an extremum, the variation of the maximum output power with voltage is relatively small, i.e., a change on V_m has a relatively small effect on the maximum power of the cell. Therefore, considering the asymptotic behavior of the *I-V* curve at short- and open-circuit conditions, the derivative required by Equation 9 can be calculated as:

$$\left.\frac{dI}{dV}\right|_{V_m} \cong -\frac{0 - I_{sc}}{V_{oc} - 0} = -\frac{I_{sc}}{V_{oc}}. \tag{10}$$

Replacing this equation into Equations 9 and 7, the voltage and current at the maximum power point and consequently the maximum output power, are expressed as follows:

$$V_m = \frac{n\,N_s k_B T}{q}\ln\left(\frac{n\,N_s k_B T}{q\,I_o}\frac{I_{sc}}{V_{oc}}\right), \tag{11}$$

$$I_m = I_{ph} + I_o - \frac{n\,N_s k_B T}{q}\left(\frac{I_{sc}}{V_{oc}}\right), \tag{12}$$

$$P_m = \left(I_{ph} + I_o - \frac{n\,N_s k_B T}{q}\frac{I_{sc}}{V_{oc}}\right)\frac{n\,N_s k_B T}{q}\ln\left(\frac{n\,N_s k_B T}{q\,I_o}\frac{I_{sc}}{V_{oc}}\right). \tag{13}$$

These equations are used to calculate key cell parameters at the maximum power point as function of both cell temperature and irradiance, which are not necessarily given by PV manufacturers. The following expression is used to calculate the photocurrent as function of irradiance and temperature [6,8,9]:

$$I_{ph} = \frac{E}{E_{ref}}\left[I_{ph,ref} + \mu_I\left(T - T_{ref}\right)\right], \tag{14}$$

where the reference state of the cell is given by the irradiance $E_{ref} = 1000\, W/m^2$ and the temperature $T_{ref} = 298.15\,K$. In this equation, μ_I is a short-circuit current temperature coefficient (*A*/*K*) and $I_{ph,ref}$ corresponds to the photocurrent obtained from a given PV cell working at (STC) reference conditions (i.e., provided by cell manufacturers). Furthermore, Villalva et al. [6] have proposed a relationship that allows the saturation current I_o to be expressed as a function of the cell temperature. In this work, this relation is explicitly written based on cell open-circuit conditions using the short-circuit current temperature coefficient as well as the open-circuit voltage temperature coefficient, hence:

$$I_o = \frac{I_{sc,ref} + \mu_I\left(T - T_{ref}\right)}{\exp\left[\frac{q\left(V_{oc,ref} + \mu_V\left(T - T_{ref}\right)\right)}{n N_s k_B T}\right] - 1}, \tag{15}$$

where $V_{oc,ref}$ is the reference open-circuit voltage and μ_V is an open-circuit voltage temperature coefficient (*V*/*K*). Finally, the quality factor of the diode *n*, which is usually considered as a constant [6,14,18], is determined at the reference state. Using the maximum power point current equation (Equation 7) and the saturation current at the reference temperature given by Equation 15, the diode quality coefficient is determined as:

$$n = \frac{q\left(V_{m,ref} - V_{oc,ref}\right)}{N_s k_B T_{ref}} \frac{1}{\ln\left(1 - \frac{I_{m,ref}}{I_{sc,ref}}\right)}, \tag{16}$$

where $V_{m,ref}$, $V_{oc,ref}$, $I_{m,ref}$ and $I_{sc,ref}$ are key cell values obtained under both actual cell temperature and solar irradiance conditions, usually provided by manufacturers.

The model is now completely determined; it requires the actual cell temperature, the actual solar irradiance and common data provided by manufacturers. The cell temperature, however, is difficult to be established; applying the energy balance equation to a module at actual and NOCT conditions, Duffie and Beckman [22] proposed a formulation for estimating the temperature as a function of solar irradiance, and an overall convective and radiation heat transfer coefficient from the cell to the environment. This coefficient is determined using a correlation that includes the wind velocity.

4. VALIDATION OF THE PROPOSED MODEL

The model has been validated against four different photovoltaic arrays given in the open literature [6,13,14]. From these works, reference data were then generated using a single-diode model which includes series and shunt resistances. Even though different approaches have been proposed to determine these data, Villalva et al. [6] based their method on a pair of series and parallel resistances satisfying the maximum power calculated by the model at STC from manufacturers' datasheet values. They have made it possible matching the

P-V curve in addition to the *I-V* curve to experimental data; in their work however, an arbitrary value of the quality factor of the diode is chosen to adjust the model. They obtained a good accuracy with respect to experimental data for different levels of irradiances and temperatures. A system of three equations with three unknowns, written from the derivatives of the *I-V* curve at short-circuits and maximum power points as well as the maximum power point current equation, was numerically solved by Sera et al. [13]. Carrero et al. [14] have assumed an ideal value of unity for the quality factor while series and parallel resistances were determined using the results of several simulations. Moreover, they mentioned that to precisely determine the resistances, complex calculation procedures are required.

To validate the proposed model, a reference numerical model is used and the calculations are performed using single-diode equations. Implicit equations for the open-circuit voltage, the short-circuit current, the maximum power point voltage and current, are solved with Newton-Raphson's method. Photocurrent and saturation current are calculated using Equation 14 and 15. The values of the resistances and the quality factor of the diode determined from different methods [13,14] are then used to solve the reference model. Table 1 summarizes calculated reference values of the resistances and the quality factor of the diodes. It is observed that higher values of n are obtained with the proposed method. Since effects of series and parallel resistances are included in reference calculations, n does not necessarily have a physical meaning. In fact, for the SP 150 array, a value higher than two of the quality factor does not necessarily mean that multi-recombination or multi tunneling steps occur, as previously discussed [20]. Therefore, the reader should be careful in attaching a particular physical meaning to the quality factors given in Table 1.

Table 1. Principal characteristics of PV arrays used for performing model validation.

Reference	Array	N_s	Reference values			Present study (Eq. 16)
			R_s (Ω)	R_{sh} (Ω)	n	n
[6]	KC 200 GT	54	0.221	415.405	1.3	1.8199
[13]	BP MSX 120	72	0.47	1365	1.397	1.8003
[14]	SP 150	72	0.932	248.2	1.0	2.0467
[14]	BP 5170 S	72	0.584	1946	1.0	1.5392

Furthermore, the proposed model has also been compared with the aforementioned reference calculations for a wide range of solar irradiances (i.e., from 50 to 1000 W/m²) and cell temperatures (i.e., from -10 to 80°C, which are representative of different climatic conditions and periods of the year [23]). Differences on currents and voltages of key PV parameters at maximum cell power conditions between the reference cases and the present method were investigated. To this end, standard mean and weighted relative errors were evaluated respectively as:

$$\varepsilon_{i,j} = \frac{\left|X_{i,j} - \bar{X}_{i,j}\right|}{\bar{X}_{i,j}} \times 100, \tag{17}$$

$$\varpi\varepsilon_{i,j} = \bar{X}_{i,j} \times \varepsilon_{i,j}, \tag{18}$$

where $X_{i,j}$ represents cell variables evaluated at given a temperature i and at a solar irradiance j, while $\bar{X}_{i,j}$ are reference values calculated iteratively under the same cell operating conditions. Estimated errors for each case studied are given in Table 2. It is observed that among characteristic key values, the maximum errors correspond to the SP 150 array (Table 2). The reference [14] shows, however, that for this particular PV array, the series resistance is quite high while the shunt resistance is relatively low. These values indicate that the corresponding array is the poorest one as compared to the others. Since the proposed methodology is based on a PV model that does not include effects due to electrical resistances, the observed differences are not a surprise. Nevertheless, they are relatively small while the proposed model could be a must for design engineers. Further, the worst array used to validate the model is characterized by the highest value of the diode quality factor, *n* (Table 1).

Table 2. Comparison of the proposed model with reference cases (Standard and weighted mean errors of different key cell parameters).

Reference case	Type of error	V_{oc} Error (%)	I_{sc} Error (%)	V_m Error (%)	I_m Error (%)	P_m Error (%)
[6]	Standard	2.06	0.05	4.66	1.08	4.51
	Weighted	1.92	0.05	4.41	0.78	2.94
[13]	Standard	1.60	0.03	3.86	1.05	3.35
	Weighted	1.51	0.03	3.73	0.48	2.35
[14]	Standard	3.75	0.38	8.18	5.27	6.01
	Weighted	3.59	0.38	8.01	1.90	4.22
[14]	Standard	2.17	0.03	4.81	1.43	5.30
	Weighted	2.06	0.03	4.68	1.47	3.61

Since the proposed model is written based on the derivative of the *I-V* curve at the maximum power operation point, the effect of this derivative is also investigated. Values obtained with the proposed method are compared to real values also determined from the derivative of the *I-V* curve at actual V_m and I_m conditions by using the implicit set of equations. Further, a standard mean error of 7.67 % is obtained between the derivative of the *I-V* curve at the maximum power point for the present model and the similar one for the third reference case (i.e., the poorest array). Note that this error is not shown in Table 2; further, for the other cases this error is much smaller. Thus, performing the derivative at the maximum power point to evaluate the voltage at this point seems to be an acceptable hypothesis.

5. ANALYSIS AND DISCUSSION

The characteristic *I-V* curves obtained by using iterative calculations as well as the present model for the KC200GT array [6] are plotted in Figures 3 to 7. The results for a constant temperature of 25°*C* and for solar irradiances of 200 W/m² and 800 W/m² are shown in Figures 3 and 5, respectively. Similar data obtained for a constant solar irradiance of 1000 W/m² and for cell temperatures of 10 and 50°*C* are illustrated in Figures 4 and 6, respectively.

From Figures 3 to 6, it is apparent that the temperature essentially affects the voltage while the current seems to be mostly affected by the irradiance. It is obvious that for high solar irradiances the proposed model is quite accurate. However, the open-circuit voltage at low solar irradiance, as shown in Figures 3 and 5, is underestimated. This seems to be principally due to the high value for the quality factor used in the present study (Table 1). In turn, series and parallel resistance do not strongly affect the behavior of both *I-V* and *P-V* characteristic curves. In particular, the temperature has a relative small effect on both the *I-V* and *P-V* characteristic key points of the solar array, especially under short- and open-circuit conditions.

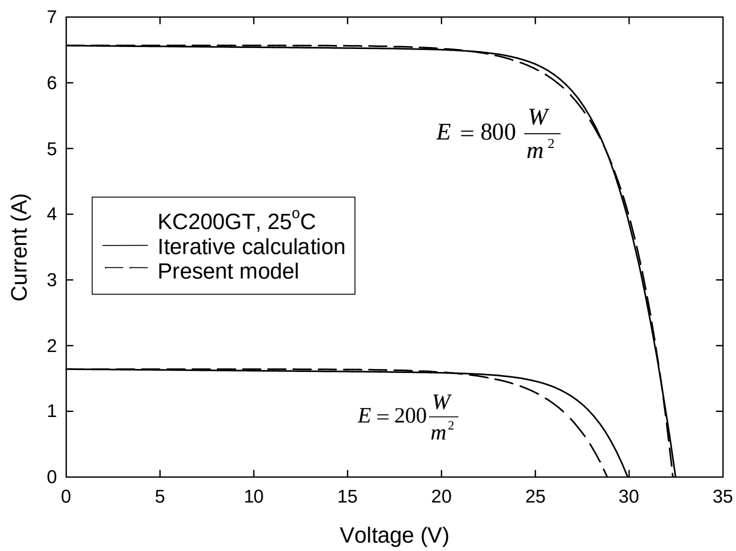

Figure 3. *I-V* curve of the KC200GT array at T=25°C.

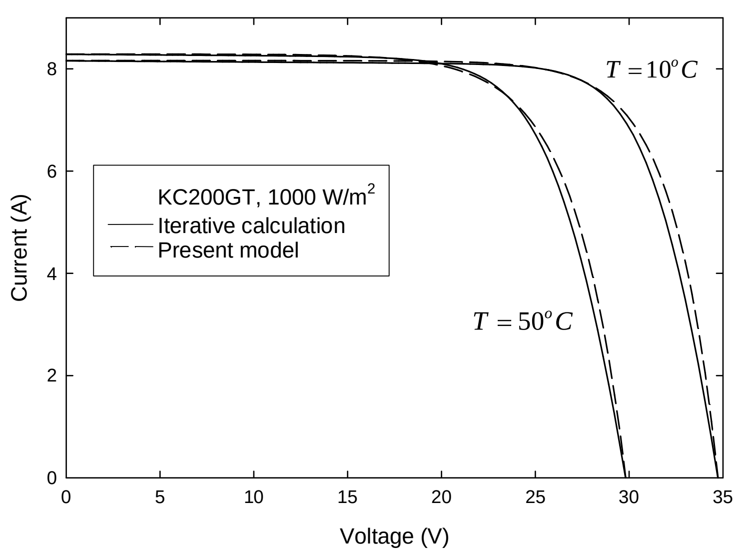

Figure 4. *I-V* curve of the KC200GT array at E=1000 W/m².

The determination of the voltage at maximum power based on the slope of the *I-V* curve using the short-circuit current and the open-circuit voltage (Equation 10), appears to be quite accurate for constant solar irradiance, as shown in Figures 4 and 6. In turn, the influence of series and parallel resistances clearly appears in these figures, in voltage- and current-source regions, respectively, where the effect of series resistance is more apparent despite the low value of the parallel resistance.

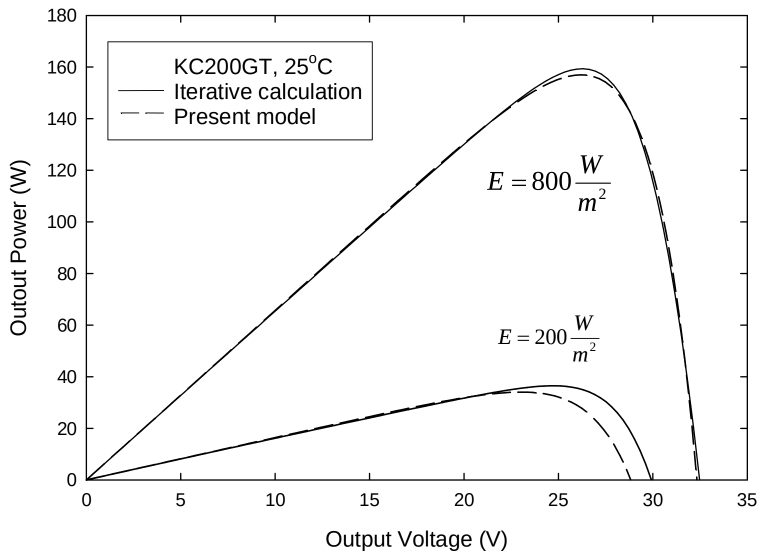

Figure 5. *P-V* curve of the KC200GT array at *T*=25°*C*.

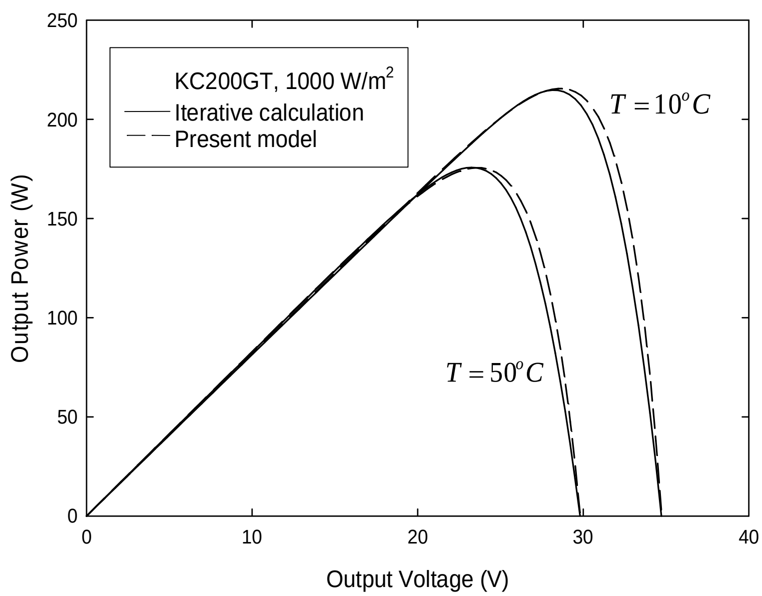

Figure 6. *P-V* curve of the KC200GT array at *E*=1000 W/m^2.

Moreover, the results of the present model can be further investigated by use of the fill factor of the PV module, defined as:

$$FF = \frac{V_m I_m}{V_{oc} I_{sc}}. \tag{19}$$

This factor represents the ratio of the power produced at the maximum power operation conditions to the maximum theoretical power that should be possible to extract from a PV module. It corresponds to the square shape of the *I-V* characteristic curve (Figure 2). Due to its simultaneous dependence on key solar cell parameters, this factor appears to be appropriate to study the performance of the present model. Green [24] has proposed four semi-empirical expressions for this factor accordingly to suitable assumptions, i.e., FF_o where the resistances are included, FF_{Rs} when only the series resistance is taken into account, FF_{Rsh} when only the parallel resistance is considered and F_{RsRsh} when both resistances are included. These factors are respectively given by:

$$FF_o \frac{v_{oc} - \ln(v_{oc} + 0.72)}{v_{oc} + 1} \tag{20}$$

$$FF_{Rs} = FF_o(1 - r_s) \tag{21}$$

$$FF_{Rsh} = FF_o\left[1 - \frac{(v_{oc} + 0.7)}{v_{oc}}\left(\frac{FF_o}{r_{sh}}\right)\right] \tag{22}$$

$$FF_{RsRsh} = FF_o(1 - r_s)\left[1 - \frac{(v_{oc} + 0.7)}{v_{oc}}\left(\frac{FF_o(1 - r_s)}{r_{sh}}\right)\right] \tag{23}$$

where $v_{oc} = V_{oc}/(n\,N_s k_B T/q)$, $r_s = R_s/(V_{oc}/I_{sc})$ and $r_{sh} = R_{sh}/(V_{oc}/I_{sc})$ are the normalized open-circuit voltage, series resistance and parallel resistance, respectively. These expressions are quite accurate for $v_{oc} < 10, r_s < 0.4$ and $r_{sh} > 2.5$ [24]. Except for the resistance terms, these relationships are only functions of the open-circuit voltage. Since both the short-circuit current and the maximum current at maximum power operation conditions increase linearly with irradiance, the differences between the open-circuit and the maximum power point voltages are more relevant.

To better validate the proposed model, Equations 20 to 24 have been examined to illustrate the influence of series and parallel resistances on the overall behavior of solar cells. Hence, fill factors calculated with Equations 19 to 23 by using values obtained with the reference model are compared to the proposed one determined with Equation 19 in Figures 7 and 8. Note that these figures show the variation of the fill factor as a function of the temperature at low irradiance and as a function of the irradiance at the reference temperature. Despite the apparent large difference observed between fill factors calculated using reference values and those calculated with the proposed model, relatively small standard mean and weighted mean errors (2.71 % and 2.66% respectively) are obtained for the KC200GT array. Figure 7 shows that when the temperature changes, the six fill factors follow almost the same trends, i.e., they decrease with increasing temperature. In turn, the slopes of FF_o, FF_{Rs} and that of the factor obtained with the present model are higher than the slopes of FF_{Rsh}, FF_{RsRsh} and the similar value obtained using the reference model. These trends indicate that the parallel resistance seems to be the principal variable that affects the performance of the PV array.

FF_o increases at a low rate with increasing irradiance (e.g., an increase of irradiance from 50 to 1000 W/m² increases the fill factor only by 0.03). In turn, when the series resistance is included in the model, (i.e., considering FF_{Rs}) a maximum is observed at about 200 W/m². As previously mentioned, the expressions given by Green [24] mainly emphasize the difference between open-circuit and maximum power point voltages.

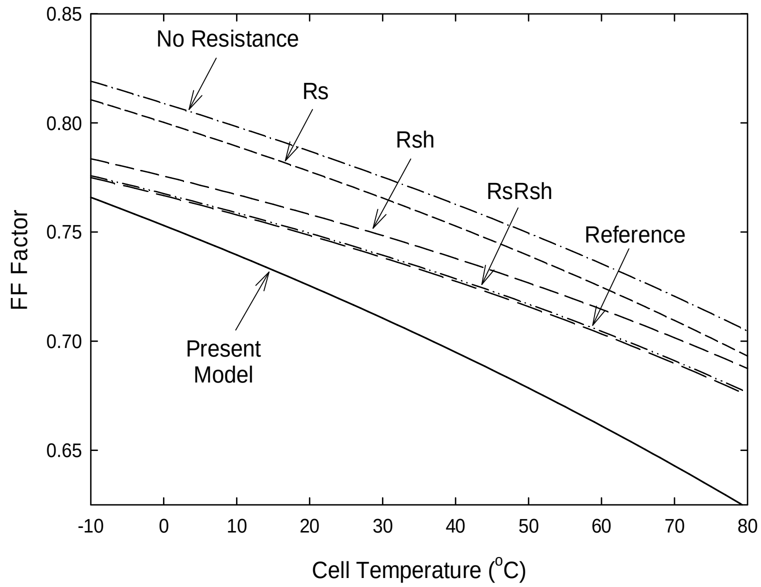

Figure 7. Fill factors of the KC200GT array at E=200W/m².

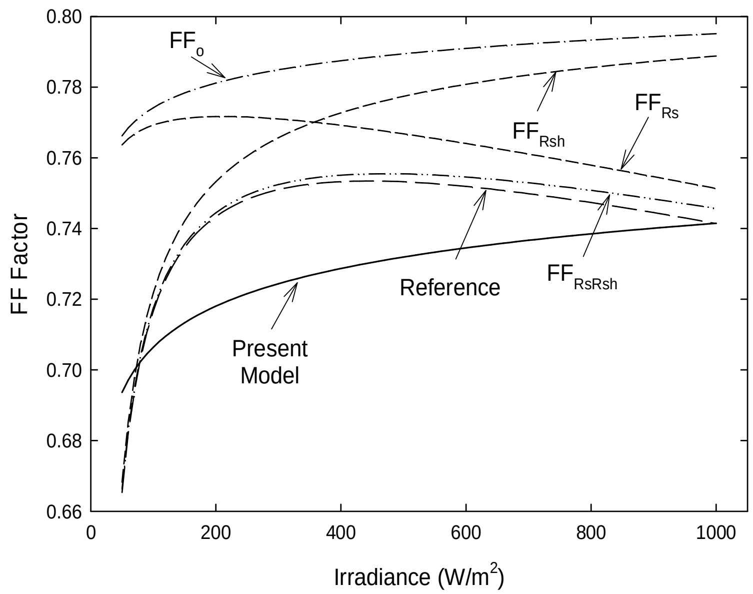

Figure 8. Fill factors of the KC200GT array at T=25°C.

Even if both of these voltages follow similar trends as a function of the irradiance, including a series resistance into the model brings about a maximum of FF_{Rs} and consequently a minimum of the difference $V_{oc} - V_m$ at low irradiance; such a behavior has been already pointed out among others by Bätzner et al. [20] and Stamenic et al. [25]. Further, since the fill factor decreases quite linearly with irradiance, then V_{oc} must increase at a higher rate than V_m.

This asymptotic behavior of FF accounts for internal losses caused by the series resistance under high irradiance conditions. They have been investigated experimentally for three c-Si modules by de Cueto [26] who approximated them at high irradiance as $FF_{loss} = -R_s I_{sc} / V_{oc}$. Under low irradiance conditions these losses (i.e., due the series resistance) are quite low; this is mainly due to relatively low currents produced under such conditions [27]. Instead, if only the parallel resistance is taken into account, (i.e., considering FF_{Rsh}), the decrease of the fill factor at low irradiance is substantial [27]. Thus, by considering both resistances, FF_{RsRsh}, at low irradiance the fill factor will be dominated by the shunt resistance and the opposite occurs for the effect of the series resistance. These analyses show that the results obtained with the reference model correspond to the evolution of FF_{RsRsh}.

Finally, the fill factor of the present study is compared to all the other fill factor expressions. Its trend is similar to that of FF_o although the values of the present study fill factor are lower. Actually, the value of the quality factor of the diode, *n*, is the only difference between these two expressions, which yields a translation of the curve and limits the value of the fill factor. Indeed, the curve of the present model coincides with the reference model at STC state, due to the fact that *n* is calculated under these conditions.

Even if the trends as a function of the irradiance are not exactly the same due to the effect of series and parallel resistances, the fill factor calculated with the proposed model appears to be a very good approximation for the actual fill factor, especially at high irradiances and low temperatures. Further, the present model underestimates the fill factor as compared with the reference case. Some care, however, should be taken in extending too far this analysis, because the results obtained with the reference model does not necessarily reflect the real behavior due to the fact that it uses constant values for both the resistances and the quality factor of the diode. Nevertheless, the explicit form of the present model makes it possible to easily and quickly calculate for engineering applications. In particular, it allows the overall performance of the solar array to be rapidly determined with acceptable accuracy as a function of both solar irradiance and cell temperature.

6. CONCLUSION

This paper proposed a simple model for characterizing the performance of PV arrays as a function of both solar irradiance and cell temperature. The model is based on an ideal cell where effects of series and parallel resistances are neglected. This simplification allows an analytical method to be used for determining current, voltage and power at every key operation conditions of the cell. Thus, explicit expressions are written for key cell parameters without the necessity of performing iterative numerical calculations. Some unknown parameters such as photocurrent, saturation current and diode quality factor are calculated based on data usually provided by PV panel manufacturers.

The proposed method is validated against reference values obtained from iterative calculations applied to known solar panels. The performance of the model is evaluated as a function of standard and weighted mean errors observed between reference and estimated values. In general the proposed model is able to provide quite accurate results; it is relatively simple to use and it can be very useful for design engineers to quickly and accurately determine the performance of PV arrays as a function of environmental constraints without carrying out numerical calculations.

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